FLUX DIFFUSION DURING MAGNETIC ACCUMULATION IN NARROW CAVITIES

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An equation is obtained for the flux diffusion during the compression of a uniform magnetic field in a flat gap. Calculations are made for fast and slow pumping of the cavity by the initial current and for a constant linear increase and an increase proportional to $\sqrt{t}$ in the initial current. It is shown that the flux losses are considerable even for large magnetic Reynolds numbers; the flux losses depend essentially on the pumping time and depend little on the shape of the pumping current pulse.

1. The compression of magnetic flux within a conducting circuit is called magnetic accumulation. Here the induction L of the circuit decreases while the current I and the magnetic field B increase. The energy of the magnetic field is

$$U=\frac{1}{2}LI^2=\frac{L_0}{L}\left(\frac{LI}{L_0I_0}\right)^2=\frac{1}{2}L_0I_0{}^2=\lambda F^2U_0$$

Here L_0 is the initial induction of the circuit, I_0 is the initial current in it, $\lambda = L_0/L$ is the turning coefficient of the circuit, and $F = LI/L_0I_0$ is the fraction of the magnetic flux remaining in the circuit.

The energetic possibilities of magnetic accumulation are limited by the fraction F of flux retained in the circuit, in connection with which the study of flux losses represents an important problem in the analysis of the operation of powerful magnetic accumulation devices, the so-called MA generators.

A large number of problems on the diffusion of a magnetic field into a stationary conductor are presented in [1]. The solutions of these problems are carried over to magnetic accumulation through the calculation of the effective depth of the skin layer and the use of the result on the conservation of the sum of the flux in the MA generator and the conductor [2].

Among the problems of magnetic accumulation considered are the compression of a field by two infinite flat conductors moving toward each other with constant velocity [3] and a number of self-similar axially symmetrical problems with $v = q/2\pi r$ [4]. Here the conductivity was assumed to be constant. It is shown that toward the end of the compression both in the plane and the axially symmetrical problems all the flux passes into the conductors.

2. Among the different types of MA generators, the flat generators]5,6] are distinguished by simplicity of construction and good energetic characteristics.

By neglecting the nonuniformity of the field in the cavity of a flat MA generator and the flux losses at the site of encounter of the walls of the cassette with the bus bars, one can adopt the following model of an unprofiled flat MA generator (Fig. 1). A uniform magnetic field is compressed in the flat cavity of an ideal piston moving with the detonation velocity D between two parallel conductors. The other side of the cavity is closed by an ideal conductor, which follows from the symmetry of the construction of the generators described in [5, 6]. The uniformity of the field in the cavity permits one to consider flux leakage only in the direction perpendicular to the conductor surface. The conductors can be assumed to be unbounded since the thickness of the skin layer is small compared with the thickness of the bus bars of the generator.

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The field in the conductor at σ = const is determined by the solution of the diffusion equation [7].

$$B_t = \mu^{-1} B_{xx} \tag{2.1}$$

which satisfies the following induction equation and initial condition at the boundary x = 0 of the cavity:

$$\frac{d}{dt}(1-t)B(t) = \frac{2}{\mu}(1-t)B_x|_{x=0} \tag{2.2}$$

$$B(x,\ 0) = B_0(x) \tag{2.3}$$

Here the time is relative to the time of motion of the compressing piston l_0/D, x is relative to the distance a between the bus bars, the magnetic field is relative to the field B_0 at the moment of capture of the flux, and the flux is relative to the initial flux $B_0 l_0 a$ in the cavity.

The parameter $\mu = 4\pi\sigma a^2 D/c^2 l_0$ represents the magnetic Reynolds number. By setting

$$B_x|_{x=0} = -q(t) \tag{2.4}$$

one can find the solution of Eq. (2.1) which satisfies the conditions (2.3) and (2.4) and from it calculate the field at the conductor boundary

$$B(t) = \frac{1}{\sqrt{\pi\mu}}\int_0^t \frac{q(\zeta)\,d\zeta}{\sqrt{t-\zeta}} + \sqrt{\frac{\mu}{\pi}}\frac{1}{\sqrt{t}}\int_0^\infty B_0(\zeta)\,e^{-\zeta^2\mu/4t}d\zeta$$

By solving this integral equation relative to q(t) one can obtain the value $B_x|_{x=0}$ and after substitution into the condition (2.2) one can arrive at an equation for the field in the cavity

$$\frac{d}{dt}(1-t)B(t) = -\frac{2(1-t)}{\sqrt{\pi\mu}}\frac{d}{dt}\int_0^t \frac{B(\tau)\,d\tau}{\sqrt{t-\tau}} + \frac{2}{\sqrt{\pi\mu}}\frac{1-t}{\sqrt{t}} + f(t) \tag{2.5}$$

$$f(t) = \frac{4(1-t)}{\mu\sqrt{\pi}}\int_0^\infty B_0'\left(\zeta\frac{2\sqrt{t}}{\sqrt{\mu}}\right)e^{-\zeta^2}d\zeta \tag{2.6}$$

After integration of (2.5) with respect to time

$$1 - F = \frac{2}{\sqrt{\pi\mu}}\int_0^t \frac{1+t-2\tau}{\sqrt{t-\tau}}B(\tau)\,d\tau - \frac{4\sqrt{t}}{\sqrt{\pi\mu}}\left(1-\frac{t}{3}\right) - \int_0^t f(\tau)\,d\tau \tag{2.7}$$

Here the flux in the cavity F = (1-t)B(t) is introduced. By multiplying the identity

$$1 = \frac{1}{\pi}\int_\tau^t \frac{d\zeta}{\sqrt{(t-\zeta)(\zeta-\tau)}}$$

by $(1+t-2\tau)B(\tau)$ and integrating it with respect to τ from 0 to t we obtain

$$\int_0^t\left(1+\frac{t}{2}-\frac{3}{2}\tau\right)B(\tau)\,d\tau = \sqrt{\frac{\mu}{\pi}}\sqrt{t} + \left(t-\frac{t^2}{4}\right) + \sqrt{\frac{\mu}{\pi}}\int_0^t f(\tau)\times$$
$$\times\sqrt{t-\tau}\,d\tau - \frac{1}{4}\sqrt{\frac{\mu}{\pi}}\int_0^t\frac{1+t-2\tau}{\sqrt{t-\tau}}B(\tau)\,d\tau - \frac{1}{4}\sqrt{\frac{\mu}{\pi}}(1-t)\int_0^t\frac{B(\tau)\,d\delta}{\sqrt{t-\tau}}$$

After double differentiation with respect to time we obtain with the help of (2.5) and (2.7) the equation for the flux in the cavity

$$\mu(1-t)\frac{d^2F}{dt^2} - \left(\frac{\mu}{2}+4(1-t)\right)\frac{dF}{dt} - 2F = \varphi(t) \tag{2.8}$$

$$\varphi(t) = 2(1-t) - 2\sqrt{\frac{\mu}{\pi}}\frac{1-t}{\sqrt{t}} - \frac{\mu}{2}f(t) + \mu(1-t)f'(t) - 2\sqrt{\frac{\mu}{\pi}}(1-t)\frac{d}{dt}\int_0^t\frac{f(\tau)\,d\tau}{\sqrt{t-\tau}} \tag{2.9}$$

The initial conditions are

$$F(0) = 1, \qquad F'(0) = f(0) \tag{2.10}$$

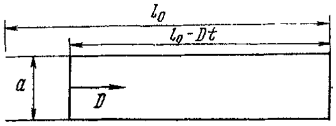

Fig. 1

The latter condition is obtained from (2.5) with the limiting transition $t \to 0$, $B(t) \to 1$.

With the change in the variable $1 - t = x^2$ Eq. (2.8) is reduced to

$$\frac{\mu}{4}\frac{d^2F}{dx^2} + 2x\frac{dF}{dx} - 2F = \psi(x) \tag{2.11}$$

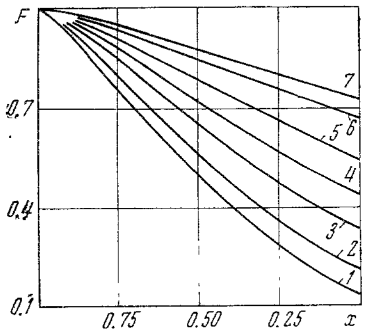

Fig. 2

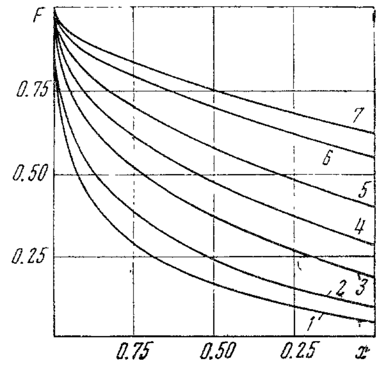

Fig. 3

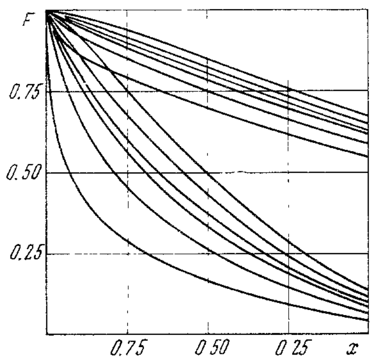

Fig. 4

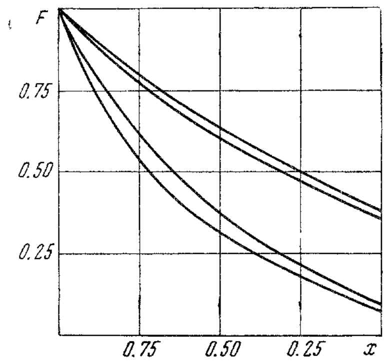

Fig. 5

the solution of which has the form

$$F(x) = F_* e^{-4x^2/\mu} + F_*' x + F_* 2\sqrt{\pi/\mu}\, x\Phi(2x/\sqrt{\mu}) + F_1(x) \tag{2.12}$$

Here $\Phi(z)$ is the probability integral and $F_1(x)$ is the partial solution of the nonhomogeneous equation (2.11) satisfying the null conditions at $x = 0$. It is easy to write $F_1(x)$ in quadratures and, using (2.6) and (2.9), to show that $F_1(x) = 0(x^4)$ as $x \to 0$, i.e., at the end of the compression the flux in the cavity is

$$F(x) = F_*' + F_* x + 4\mu^{-1} F_* x^2 + 0(x^4)$$

Thus, toward the end of the compression two constants (F_*, F_*') which determine the flux leakage are generated from the initial distribution of the field in the walls of the cavity.

3. In the experiments the initial field in the generator cavity is produced by a current source which pumps the generator. The initial field distribution $B_0(x)$ in the cavity walls depends on the pumping time t_0 and the shape of the pumping current pulse and is determined by the solution of Eq. (2.1) for the given law of variation of the field at the conductor boundary.

Two limiting cases of pumping can be distinguished: slow pumping ($T_0 \to \infty$) when the field penetrates deep into the cavity walls and $B_0(x) = 1$, and rapid pumping ($t_0 \to 0$) when the field does not penetrate into the conductor and $B_0(x) = 0$ while at the boundary $B_0(o) = 1$. With slow pumping the solution of Eq. (2.8) has the form

$$F_s(t) = 2\left(1 - \frac{\mu}{8}\right) B_*(t) + \frac{\mu}{4} - \sqrt{\frac{\mu}{\pi}}\sqrt{t} + \frac{8(1-t)}{\sqrt{\pi\mu}} \arcsin\sqrt{t} -$$

$$- 2\sqrt{1-t} + (1-t) + 4\sqrt{\frac{\pi}{\mu}}\left(1 - \frac{\mu}{8}\right)\sqrt{1-t}\,(1 - B_*(t))$$

$$B_*(t) = e^{4t/\mu}\,(1 - \Phi(2\sqrt{t}/\sqrt{\mu}))$$

The function $B_*(t)$ describes the decrease in the field in a stationary flat gap bounded by a conductor without a field.

At the end of the compression with slow pumping the flux remaining in the cavity is

$$F_{s*} = \mu/4 - \sqrt{\mu/\pi} + 2(1-\mu/8) B_*(1)$$
$$F_{s*} \approx 1 - \frac{16}{3\sqrt{\pi}}\frac{1}{\sqrt{\mu}} + \frac{6}{\mu} - \ldots \quad \text{for} \quad \mu \gg 1$$
$$F_{s*} \approx \frac{\mu}{4} + \frac{1}{4\sqrt{\pi}}\mu^{3/2} + \frac{1}{16\sqrt{\pi}}\mu^{5/2} - \ldots \quad \text{for} \quad \mu \ll 1$$

In the case of rapid pumping

$$F_r(t) = \left(1+\frac{8}{\mu}\right) B_*(t) - \frac{4}{\sqrt{\pi\mu}}\sqrt{t} - \frac{8}{\mu}\sqrt{1-t} + \frac{4(1+8/\mu)}{\sqrt{\pi\mu}}\sqrt{1-t}\arcsin\sqrt{t} - 2\sqrt{\frac{\pi}{\mu}}\left(1+\frac{8}{\mu}\right)\sqrt{1-t}\,(1-B_*(t))$$

At the end of the compression

$$F_{r*} = (1+8/\mu) B_*(1) - 4/\sqrt{\pi\mu}$$
$$F_{r*} \approx 1 - 8/\sqrt{\pi\mu} + 12/\mu - \ldots \quad \text{for} \quad \mu \gg 1$$
$$F_{r*} \approx \frac{1}{8\sqrt{\pi}}\mu^{3/2} - \frac{3}{32\sqrt{\pi}}\mu^{5/2} + \ldots \quad \text{for} \quad \mu \ll 1$$

The ratio of flux losses in the two cases of pumping with $\mu \gg 1$ is

$$(1 - F_{r*}) / (1 - F_{s*}) = {}^3/_2$$

The dependence F(x) for μ = 1, 2, 5, 10, 20, 50, and 80, corresponding to curves 1-7, is presented in Fig. 2 (slow pumping) and Fig. 3 (rapid pumping).

The rapid pumping leads to the greatest flux losses and the slow pumping to the least. The other cases of pumping by a current increasing with time give

$$F_{r*} \leqslant F_* \leqslant F_{s*}$$

Numerical calculations were made of the flux diffusion during pumping with a constant current, with a current increasing in proportion to $\sqrt{t}$ and with a linearly increasing current. The calculation was conducted in two stages. First the partial solution $F_1(x)$ of the nonhomogeneous equation (2.11) with null conditions at x = 0 was sought by the standard Runge–Kutta program with automatic choice of the step and only the values F_1 and F_1' at the point x = 1 were derived. Then by substituting these values into the general solution (2.12) the constants F_* and F_*' were determined from the initial conditions (2.10), and Eq. (2.11) was solved again with $F(0) = F_*$ and $F'(0) = F_*'$. For all the cases of pumping examined $F_*' = 2\sqrt{\pi/\mu}F_*$.

The dependence of the flux losses on the pumping time is shown in Fig. 4 where graphs are presented for μ = 1 (lower group of curves) and μ = 50 (upper group of curves) and $t_0 = 10^{-8}$, 0.1, 0.5, 1, 5, and 10^4 with pumping by a constant current. The largest value of t_0 corresponds to the top curve in Fig. 4.

The effect of the shape of the pumping pulse on the flux losses is illustrated in Fig. 5. The curves presented in it pertain to t_0 = 1 and μ = 1 (two lower curves) and μ = 10 (upper curves) with pumping by a constant current (upper of two adjacent curves) and by a linearly increasing current (lower curves). For pumping with a current increasing in proportion to $\sqrt{t}$ the graph of F(x) is located between the curves of pumping with a linear and a constant current.

The flux losses depend weakly on the shape of the pumping current pulse and are determined mainly by the magnetic Reynolds number μ and the pumping time t_0.

In the model examined the flux losses proved to be considerable even for large μ. For example, $0.23 \leq 1 - F_* \leq 0.45$ at μ = 50 and $0.56 \leq 1 - F_* \leq 0.72$ at μ = 10. Such losses are connected with the compression of the magnetic field in the cavity and with the strong increase in the field gradient at the conductor boundary.

The model of flux losses described can be used for estimates of this parameter for flat MA generators and must be improved for an examination of the last stages of the compression when the geometry of the field compression changes and the problem becomes close to that described in [3].

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